



Nonlinear Acoustic Waves: Modeling - Numerics - Optimization

High intensity (focused) ultrasound HIFU is used in numerous medical and industrial applications ranging from lithotripsy and thermotherapy via ultrasound cleaning and welding to sonochemistry. We will highlight certain mathematical and computational aspects related to the relevant nonlinear acoustic phenomena, namely - modeling of high intensity ultrasound phenomena as second and higher order wave equations - efficient and robust time integration - absorbing boundary conditions for the treatment of open domain problems - optimal shape design and boundary control problems in the context of HIFU. The talk is based on joint work with Rainer Brunnhuber (Alpen-Adria-Universität Klagenfurt), Christian Clason (University of Duisburg-Essen), Manfred Kaltenbacher (TU Vienna), Irena Lasiecka (University of Memphis), Vanja Nikolic (TU München), Petronela Radu (University of Nebraska at Lincoln), Gunther Peichl (University of Graz), Igor Shevchenko (Imperial College London), and Mechthild Thalhammer (University of Innsbruck).

Physical and mathematical modeling of high intensity ultrasound and generally of nonlinear acoustics is still an ongoing process and a field of active research. The classical models of in this context are nonlinear wave equations exhibiting potential degeneracy as well as strong damping. Taking into account higher order effects leads to third or fourth order PDEs. An additional important issue is the coupling of acoustics to other physical fields, e.g., when focusing by a linearly elastic silicone lens immersed in the nonlinearly acoustic fluids, as typical for a widely used class of lithotripsy devices. In the simulation of high intensity ultrasound, a particular challenge due to nonlinearity and the presence of different wave lengths is efficient and robust time integration. For this purpose, a promising approach are operator splitting techniques exploiting the intrinsic structure of the equations. The original second or third order in time evolution equations are split into simple subproblems that can be solved by standard methods or even explicitly. Combination of such appropriately weighted subproblems by exponential operator splitting schemes leads to highly efficient time integration methods. Strictly speaking, acoustic sound propagation takes place in full space, or at least in a domain that is typically much larger than the region of interest. To restrict attention to a bounded domain, e.g. for computational purposes, artificial reflections on the boundary have to be avoided. This can be done by imposing so-called absorbing boundary conditions ABC that induce dissipation of outgoing waves. Here it will turn out to be crucial to take into account nonlinearity of the PDE also in these ABC. Finally, we will discuss some practically relevant optimization problems in the context of nonlinear acoustics applications in lithotripsy. The optimal choice of ultrasound excitation via piezoelectric transducers leads to a boundary control problem; focusing high intensity ultrasound by a silicone lens requires shape optimization. For both problem classes, we will discuss the derivation of gradient information in order to formulate optimality conditions and drive numerical optimization methods.