



Splitting methods: analysis and applications

Splitting methods are a well-established tool for the numerical integration of time-dependent partial differential equations. The basic idea behind splitting is to decompose the vector field into disjoint parts, integrate them separately (with an appropriate time step), and finally combine the single flows in the right way to obtain the sought-after numerical approximation. My lecture will consist of four parts; each of them will take approx. 90 minutes.

1. Introduction to splitting methods. The first lecture is devoted to a general introduction to splitting methods and their convergence properties for linear problems. The numerical analysis will be carried out in an appropriate functional analytic framework. Here, the linear Schrödinger equation serves as a typical example.

2. Splitting for the Vlasov equation. The Vlasov equation (in combination with the Poisson or the Maxwell equations) is a well established model in plasma physics. The state space of this time-dependent problem is six dimensional, which makes the application of standard integration schemes impossible. With the help of this equation, it will be demonstrated how splitting methods can be efficiently applied to and analyzed for nonlinear problems.

3. Some dispersive problems. A splitting approach for the Korteweg-de Vries (KdV) and the Kadomtsev–Petviashvili (KP) equation with periodic boundary conditions will be considered. The Burgers term in these equations will be treated with the method of characteristics. A similar approach is possible for the magnetic Schrödinger equation.

4. Splitting in the presence of boundary conditions. For diffusion-reaction equations, splitting potentially offers computational efficiency. However, when the problem is equipped with nontrivial boundary conditions, splitting methods usually suffer from order reduction and some additional loss of accuracy. In this lecture a modification of the classic splitting schemes will be proposed that resolves the problem of order reduction in the case of oblique boundary conditions.